The Border Effect on Prices: The Role of Local Product Availability*

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Abstract

While the impact of borders on price convergence for traded goods is well established, the role of regional disparities in product variety remains underexplored. We develop a model in which local products—those sold exclusively within a single country—emerge endogenously. Surprisingly, in a symmetric setting, border costs for local products are *lower* than those for traded goods, which are available in multiple countries. The presence of local products drives divergence in traded good prices. Borders play a limited role in determining whether a product is local or traded; instead, firms make this decision based on product's fixed costs. We then show that failing to account for local products leads to biased estimates of the border effect. Finally, we discuss empirical strategies to properly control for the presence of local products.

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1 Introduction

We examine the impact of competition from local products on the pricing decisions of traded products in two regions separated by a border. We show that border costs are unrelated to the decision of products to be traded. Specifically, whether products are local or traded depends on fixed production costs, such as distribution costs or brand positioning. We demonstrate that products are traded even when border costs are prohibitively high in terms of the model. Our model disentangle the role of fixed costs, which determines whether products are traded, from border costs that impact pricing by shifting demand.

We provide a simple extension of the Hotelling (1929) model to demonstrate that the border effect will be biased if local goods are not controlled for. First, we provide a benchmark for how borders lead to price dispersion among traded products in a setting that is otherwise symmetrical. This is the isolated effect of borders on the pricing decision for traded goods. Even when border costs are high, products are traded; i.e., sold in both countries. Nevertheless, border costs will affect prices by shifting demand between countries. Next, keeping our model simple, we add a local brand in one of the political regions and estimate price dispersion.[1] A local product is sold only on one side of the border, regardless of the demand for it on both sides. We found that dispersion increases from the estimated border due to local competition. We show that traded and local products arise in equilibrium. The key for the results is that fixed costs being high enough. If fixed costs are zero, there are no local products and all goods are traded. Also, if fixed costs are too high, then there could be no local products.

Next, we address the influence of local products on regression discontinuity (RDD) estimation, a widely used technique in empirical border research. A fundamental requirement of RDD is that all variables, other than the treatment, exhibit continuity at the cutoff. However, local products, by their very nature, are discontinuous, existing on only one side of the border. This discontinuity creates a confounding effect, obscuring the true impact of the border if not accounted for. Our model demonstrates that local product are determined by fixed production costs rather than border-related costs, so they should be controlled for in the estimations. Finally, we discuss various methods employed to address and correct the bias introduced by local products in border effect estimations.

Borders between regions or countries have been one of the most extended explanations for the non-convergence of prices in the trade literature (Engel and Rogers (1996); Anderson and van Wincoop (2001); McCallum (1995); Parsley and Wei (2001); Gorodnichenko and Tesar (2009); and more recently Gopinath, Gourinchas, Hsieh, and Li (2011); Beck, Kotz, and Zabelina (2020); and Messner, Rumler, and Strasser (2024)). Additionally, trade costs play a significant role in explaining how goods are traded between different

[1]Political regions can be countries, states, counties, or neighborhoods.

markets (Anderson and van Wincoop (2003), Anderson and van Wincoop (2004), Atkin and Donaldson (2015), Auer, Burstein, and Lein (2021), and Burstein, Lein, and Voguel (2024)).[2]

What is a border? Engel and Rogers (1996) defines a border as the political frontier between the US and Canada. In gravity trade models, borders conceal—at least—three different factors: tariffs, language, and currencies. While the first impacts prices directly, the other two are fixed costs that consumers must pay to arbitrage trade, and as such, they shift demand. Analyzing trade between countries Head and Mayer (2014) reveals that borders have significant effects even when tariffs are small. They also conclude that language and currency differences between countries appear to have a more substantial impact than reasonable (p. 189). Nevertheless, in the price dispersion literature, borders remain political frontiers between nations or states within countries.

Geographical regions differ in consumers' income and preferences for products. For example, Bronnenberg, Dube, and Gentzkow (2012) demonstrated that preferences are geographically based and, more interestingly, time persistent. In such settings, producers may offer products that match local characteristics. Those idiosyncrasies may include size, flavor, brand, and presentation, among others. Consistently, the overwhelming evidence shows that most retail products are not traded, i.e., local. For example, Broda and Weinstein (2008) established that in "the typical bilateral city/region comparison between the US and Canada, only 7.5 percent of the goods are common" (page 11). Gopinath, Gourinchas, Hsieh, and Li (2011) found that only 3.4 percent of 125,048 products on their database were available in the US and Canada, even for the same retail chain. More recently, Messner, Rumler, and Strasser (2024) analyzes transactions at the border between Austria and Germany, establishing that "once we restrict the sample to products sold on both sides of the border, we are left with a tenth of products..." (page 8). Finally, Beck, Kotz, and Zabelina (2020) found that for Belgium, Germany, and the Netherlands, less than a tenth of products were in both pairs of countries. The soda market provides a clear example: despite the dominance of global players like Coke and Pepsi, they operate alongside numerous distinct local products.[3]

As a result of being such a variety of local brands, traded goods in two different markets are exposed to very different competitive conditions. While the literature has examined the effect of borders on price convergence, if local goods influence the prices of traded goods in the local market, then the impact of borders may be misestimated. While a traded product is on both sides of the border, local products vary. Many local

[2] Other explanations for price divergence include the existence of high fixed costs of production for some goods (Coşar, Grieco, and Tintelnot (2015a); Coşar, Grieco, and Tintelnot (2015b)), price discrimination of consumers (Haskel and Wolf (2001), Dvir and Strasser (2018)), a different currency (Cavallo, Neiman, and Rigobon, 2015), or—within countries—sticky prices (Crucini, Shintani, and Tsuruga (2010), Elberg (2016)).

[3] See list at Wikipedia: List of Soft Drinks by Country.

products compete with traded products at the same store, sometimes meters away on the same shelf. On the other hand, traded products may compete with each other even when they are kilometers apart.[4] In terms of gravity, it is easy to justify that local brands should affect the price of traded brands.

In Borraz and Zipitría (2022), we showed that local brands create price dispersion regardless of the border. Prices are set by stores, taking into account local competitive conditions and their product mix. The effects of local competition and local products must be distinguished from the role of international borders in the price dispersion of traded products. In other words, border costs for traded products may be eliminated, and price dispersion persists because local products affect competition in both locations differently. The paper extend this model to add a border. This simple setting enables us to disentangle the effects of local competition and fixed costs from those of borders. Why is this important? Any policy objective that aims to achieve price convergence must consider which trade barriers are relevant. Working on reducing trade costs—i.e., "borders"—that impede the movement of traded products may not be an optimal policy if other costs persist. In those cases, price dispersion has a positive lower limit.

Our paper relates to the literature that has shown ample evidence of the effect of differences in varieties on prices within and between countries. Between countries, Cavallo, Feenstra, and Inklaar (2023) and Beck and Jaravel (2021) demonstrated that differences in varieties affect the cost of living. Within countries, Handbury and Weinstein (2015) showed how the availability of different varieties across cities in the US biases the estimation of price indexes. Auer, Burstein, and Lein (2021) showed that the appreciation shock to the Swiss Franc has distinct impacts depending on the share of imported products. While the paper does not address local products, it shows that the effects of an appreciation vary according to local conditions. In Borraz, Carozzi, González-Pampillón, and Zipitría (2024), we showed that following a demand shock in Montevideo, Uruguay, stores increased the number of varieties offered, leading to a 3% price decrease. In Borraz and Zipitría (2022), we showed that differences in the number of varieties affect price convergence between stores in Uruguay. Finally, in Borraz and Zipitría (2024), we demonstrate that changes in varieties are correlated with price dispersion in both the short and long run. All these papers show evidence of varieties, primarily in local products, influencing price setting. Nevertheless, none has addressed their impact on price dispersion of traded goods between countries.

The paper is organized as follows. In the next Section, we introduce the model and show the effect of borders on price dispersion. Section 3 derives price dispersion in the presence of borders and local goods. Section 4 shows how local products bias the

[4]In the empirical analysis, the distance to the border on each side may include stores or households 60 kilometers Messner, Rumler, and Strasser (2024), 80 kilometers Beck, Kotz, and Zabelina (2020) and reach up to 500 kilometers Gopinath, Gourinchas, Hsieh, and Li (2011).

estimations in the standard RDD models used in the literature. It also provides various methods for accounting for the local product effect in the border estimation. Finally, Section 5 show the main conclusions of the paper.

2 The Border Effect

We model our economy based on Hotelling (1929). This model is well-suited for analyzing both international and local settings simultaneously and how the constellation of parameters determines whether products are local or traded in equilibrium. Most papers on price dispersion are empirical, except Gopinath, Gourinchas, Hsieh, and Li (2011). In that paper, they develop a circular city model rather than a linear one. As we introduce several parameters, this more straightforward model helps us understand the underlying economic forces that explain price dispersion. In this section, we establish the model's setup and define the border as an exogenous cost for consumers to buy goods from another country. Additionally, one of the key aspects of our model is to determine the conditions under which the pricing and choice selection of the stores constitute a Nash equilibrium. This section presents the baseline of the effect of the borders on the prices of traded goods. It is required to disentangle borders' impact on other effects, such as demand conditions or fixed distribution costs.

We assume a linear city with two stores and a continuum of consumers on a road of distance R. Consumer located at j has utility $U_j = u - t\,|x_j - x_d| - p_d$, where u is the reservation utility of the consumer located at j and equal for all consumers,[5] t is the cost per unit of distance for the consumer located at j to move to the store located at d, and p_d is the product's price at the store in d. Stores are located at $d = \{0, R\}$, i.e., at the beginning and end of the road, denoted S_0 and S_R respectively. For simplicity, assume that variable production costs are zero.[6] Nevertheless, firms must pay a fixed cost of F for each product sold, which may include expenses related to brand positioning or distribution. Although it may not be relevant here, we will revisit this assumption in Section 3.

To solve this model, we first need to find the indifferent consumer between buying in either store, denoted $\widehat{x}$. For this consumer, we have: $u - t(\widehat{x} - 0) - p_0 = u - t(R - \widehat{x}) - p_R$. As the consumer $\widehat{x}$ is indifferent between buying in either store, those to its right or left are not. They have to travel more to reach any store. Then, all consumers up to $\widehat{x}$ will buy at store S_0, and its demand will be $D_0 = \widehat{x}$, while the remaining $R - \widehat{x}$ consumers will buy at store S_R, whose demand we denote as D_R. Solving, we have that

[5] If reservation utility differs, then it will difficult to find the indifferent consumer. It is a standard assumption in the literature.

[6] This is a simplifying assumption. Lifting does not change the model's results and makes the model more difficult to interpret.

$\widehat{x} = D_0 = \frac{p_R - p_0 + tR}{2t}$. Firms maximize profits, which are $\pi_0 = \left(\frac{p_R - p_0 + tR}{2t}\right) p_0 - F$ and $\pi_R = \left(\frac{p_0 - p_R + tR}{2t}\right) p_R - F$. Maximizing prices, we obtain the reaction functions of each firm: $p_0 = \frac{p_R + tR}{2}$ and $p_R = \frac{p_0 + tR}{2}$. Equilibrium prices are $p_0 = p_R = tR$; therefore, this setting has no price dispersion. Firms have market power, as they are imperfect substitutes for consumers due to travel costs t.

Now, we introduce a cost for the consumer to cross a hypothetical border between stores.[7] We assume the border is at some place B between both stores at the right of $\widehat{x}$.[8] As in Gopinath, Gourinchas, Hsieh, and Li (2011), the border implies a fixed cost of β for consumers who cross it to buy from a store on the other side. This may be due to differences in currencies, languages, or traveling costs between countries. From now on, we will refer to stores and countries interchangeably, as each region will have only one store. The utility is now:

$$U_j = u - t\,|x_j - x_d| - \beta \times \mathbb{1}\{c_j \neq c_d\} - p_d, \tag{1}$$

Where $\mathbb{1}\{c_j \neq c_d\}$ is an indicator function that equals one if the country of the consumer j and the store d differ, and zero otherwise. With a positive border cost, if the border B is to the right of $\widehat{x}$, the indifferent consumer $\widehat{x}$ will shift to x^B, depending on the magnitude of the border cost β. We refer to x^B as the indifferent consumer between buying at either store, if $\beta > 0$. Intuitively, if some consumers in country 0 would buy in country R without the border, they would now prefer to buy in their home country due to the border cost. Figure 1 below depicts the setting. The consumers who shift stores or countries due to border restrictions are represented by the 45-degree dashed lines.

Note that if the border B is at $\widehat{x}$, the border does not play any role as it does not shift demand between locations. The following Lemma shows this result.

Lemma 1. *If the border is at the same place as the indifferent consumer, then any positive border cost is irrelevant.*

Proof. Assume two consumers, each located at a small ε distance to the border B. As the consumer at the left of b prefers to buy at S_0, then it must be that $u - t\,(B - \varepsilon) - p_0 > u - t\,[R - (B - \varepsilon)] - p_R + \beta$, and solving for $(B - \varepsilon)$ we obtain $(B - \varepsilon) > \frac{p_R - p_0 + tR}{2t} - \frac{\beta}{2t}$. For the consumer located at the right, as she prefers S_R to S_0, her utility must be such as $u - t\,(B + \varepsilon) - p_0 + \beta < u - t\,[R - (B - \varepsilon)] - p_R$, and solving for $(B + \varepsilon)$ we obtain $(B + \varepsilon) < \frac{p_B - p_0 + tR}{2t} + \frac{\beta}{2t}$. As $\varepsilon \to 0$, we obtain $\frac{p_R - p_0 + tR}{2t} - \frac{\beta}{2t} < B < \frac{p_R - p_0 + tR}{2t} + \frac{\beta}{2t}$. Then, $B = \frac{p_R - p_0 + tR}{2t} = \widehat{x}$. □

Assume that $\widehat{x} < B$, i.e., the border is to the right of the indifferent consumer. Then, for every positive border cost β, the indifferent consumer should move from $\widehat{x}$ through

[7] A similar assumption is made in Gopinath, Gourinchas, Hsieh, and Li (2011).

[8] The scenario of the border being at the left of $\widehat{x}$ is symmetrical to the one derived here.

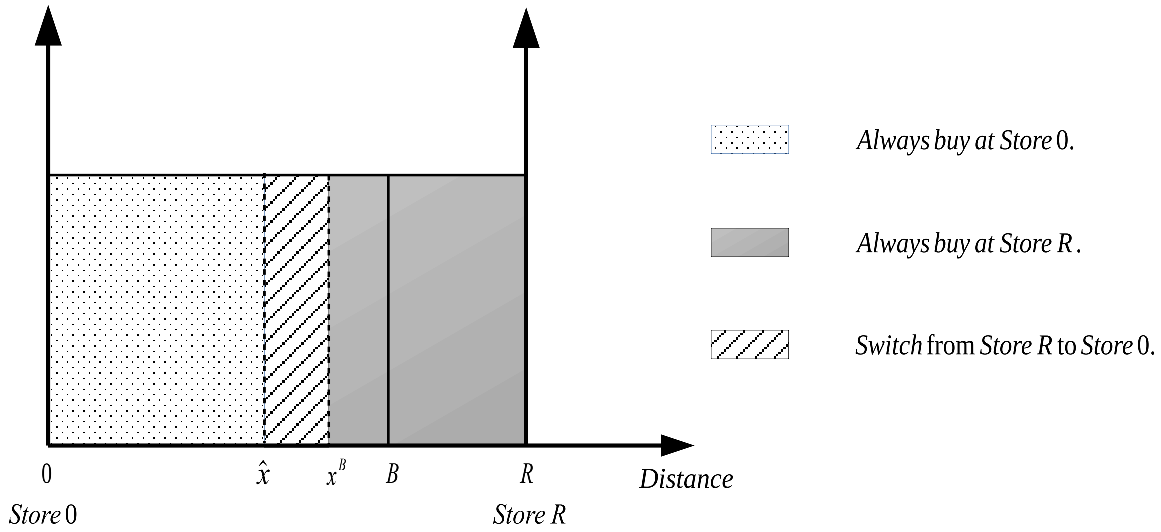

Figure 1: A Linear City with Border

x^B. The new indifferent consumer x^B should be at $\widehat{x}+\beta$, as the utility is linear in cost. As a result, $x^B = \widehat{x}+\beta = \frac{p_R - p_0 + tR}{2t} + \beta$, where $\beta \in [0,\,(B-\widehat{x})]$. The demand for store S_0 is $D_0^B = \frac{p_R - p_0 + tR + 2t\beta}{2t}$, and profits are $\pi_0^B = D_0^B \times p_0 - F = \left(\frac{p_R - p_0 + tR + 2t\beta}{2t}\right) \times p_0 - F$. Maximizing in price p_0 we obtain the reaction function is $p_0 = \frac{p_R + tR + 2t\beta}{2}$. Demand for store S_R is $D_R^B = \frac{p_0 - p_R + tR - 2t\beta}{2t}$, and profits $\pi_R^B = D_R^B \times p_R - F = \left(\frac{p_0 - p_R + tR - 2t\beta}{2t}\right) \times p_R - F$. Maximizing in price p_R we obtain the reaction function $p_R = \frac{p_0 + tR - 2t\beta}{2}$. The equilibrium prices are $p_0^B = tR + \frac{2t\beta}{3}$ and $p_R^B = tR - \frac{2t\beta}{3}$. Our second Lemma follows.

Lemma 2. *Borders make price convergence less likely.*

Proof. Now $(p_0^B - p_R^B) = \frac{4}{3}t\beta$. □

If x^B is at the left of $\widehat{x}$ instead, then the prices p_0^B and p_R^B reverse, but price difference is the same. We now compute the size of the border by substituting p_0^B and p_R^B in $x^B = \widehat{x} + \beta = \frac{\beta}{3} + \frac{R}{2}$. As x^B is at the right of $\widehat{x}$, then $x^B \in \left[\frac{R}{2},\, R\right]$. Substituting we obtain that $\beta \in \left[0,\, \frac{3}{2}R\right]$. If the border value is zero, there is no border; i.e., $x^B = \widehat{x}$. As the border cost β increases, x^B moves further to the right until it reaches B. Now, countries are in autarky, and while β could be as high as infinity, it does not change the result: markets are segmented.

Because borders shift demand, prices change in response to borders, and price convergence becomes more challenging. Our results also predict which store or country increases its market power. Store S_0 gains from having a border, as some consumers who would otherwise travel to another country now have to buy at home. This allow firm S_0 to increase its price from $p_0 = tR$ to $p_0^B = tR + \frac{2t\beta}{3}$. The reverse is true for store S_R, which loses demand due to the border cost and has to decrease its price. Also, the model predicts which consumers will cross the border. Consumers $(B - x^B)$ in country 0 cross the border and buy at store S_R. Nevertheless, the border costs allow store S_0 to capture demand $\left(x^B - \beta\right)$ that it would otherwise lose without the border. Lastly, and more importantly for Section 4, we still have traded products even if border costs β reach their maximum. A product to be traded does not depend on border costs but on the fixed costs F, as shown in Section 3 and formally in the Appendix A.

We now have a baseline of the actual costs of the border on price dispersion. This analysis has isolated the border cost from other possible explanations for price differences between countries. In the next section, we introduce demand for a second product variety that competes with the local product—i.e., other variety of soda—and analyze how the results change. We then demonstrate how price dispersion changes due to the presence of these local brands.

3 The Border with Local Products

Section 1 demonstrated that countries vary in the types of goods offered to consumers. As a result, traded brands must contend with different local brands in each country, resulting in varying levels of competition. What happens when a traded brand — the one sold in both countries — competes with a local brand? Following Borraz and Zipitría (2022), we investigate this asymmetry between countries by endogenously allowing a local product to be sold exclusively in one store, specifically in one of the countries. In that paper, we demonstrated that prices do not converge when competition conditions differ between stores, even without borders. In that paper, we showed that local brands can arise endogenously without borders. This simple setting enables us to explain how price convergence is influenced when local brands or local varieties of products are available in different markets and how this affects the estimation of the border effect. That is, we will disentangle the impact of borders on competition between traded brands from competition between traded brands and local brands at the country level. In what follows, we will interchange brands and varieties to refer to products that can be traded internationally or locally. We assume that both the local and traded products are varieties on the same product market, such as two brands of sugar, or soda.

At each point along the line, there are now two types of consumers, differing in their preference for product variety, $z_i = \{z_T, z_L\}$. While the distance dimension is continuous, variety is discrete. Furthermore, a mass $(1 - \lambda)$ of consumers prefer variety z_T, and a mass λ of consumers prefer variety z_L. The model could be represented as two lines of distance R, one on top of the other.[9] We assume no difference between countries in their preferences for goods, as we want local brands to arise endogenously in a symmetrical

[9]See Borraz and Zipitría (2022).

setting. Stores could offer varieties s_q, with $q = \{T, L\}$. If a product is sold in both countries, it is the traded brand. If it is sold in only one country, it is considered a local brand, regardless of the demand for the product in other countries. Product T is available at both stores, the traded one, but product L is a local product available only at the store S_0. That is, we are imposing some brand asymmetry between stores that affects the local pricing decisions for products on both sides of the border, in line with the literature discussed in Section 1. In Appendix A, we show that this setting is a Nash equilibrium. We maintain our assumption of a fixed cost of F for each brand sold and zero variable costs.

The consumer utility is now:

$$U_{ji} = u - \theta \times \mathbb{1}\{z_i \neq s_q\} - t\,|x_j - x_d| - \beta \times \mathbb{1}\{c_j \neq c_d\} - p_{dq},$$

Where j represents the consumer's location and i represents her brand preference, θ is a fixed cost the consumer pays if the brand available (s_q) differs from her preferred one (z_i). The utility function has three costs on consumer utility: one that lowers his utility if his preferred variety is unavailable (θ), one that taxes her for buying in another country (β), and a transport cost t for reaching the store regardless of brand preference.

In Borraz and Zipitría (2022), if there is no border (i.e., $\beta = 0$), we found that the price of the traded products are $p_{T0} = tR - \frac{\lambda\theta}{6}$ and $p_{TR} = tR - \frac{\lambda\theta}{3}$, and prices do not converge due to different competitive conditions at the store. Prices are lower than if no local product is available, as competition is higher. Nevertheless, stores are differently affected by the local product. The price difference between both products is $(p_{T0} - p_{TR}) = \frac{\lambda\theta}{6}$. In this setting, there are two indifferent consumers: the "location" consumer ($\widehat{x_T}$, similar to our previous $\widehat{x}$) who is indifferent between buying the traded product T in either store/country, and the "local" consumer ($\widehat{x_L}$), who is indifferent between buying the L local product at S_0 instead of reaching store S_L and buying the T non-preferred traded variety. We also showed that the indifferent consumer for the local variety ($\widehat{x_L}$) should be to the right of the indifferent consumer for location ($\widehat{x_T}$). The intuition is simple. The indifferent consumer of local products faces two penalties: one for purchasing the non-preferred traded brand and another for switching to another store. Then, compared to consumers who only need to move to another store to buy their preferred variety, a larger number of consumers of the local brand will stick to their preferred brand because it is unavailable at the other store.

Assume the border B is to the right of $\widehat{x_T}$ and $\widehat{x_L}$. As $x_T \neq x_L$, the effect of the border will differ for the consumers of a local product L than for consumers of the traded product T. Figure 2 below depicts the setting. Again, store S_R loses sales due to the border; some consumers prefer the trade brand T, and others like the local product L. To keep the figure simple, we show only the two border indifferent consumers, x_T and

x_L, at the exact location, whereas they should be separated.

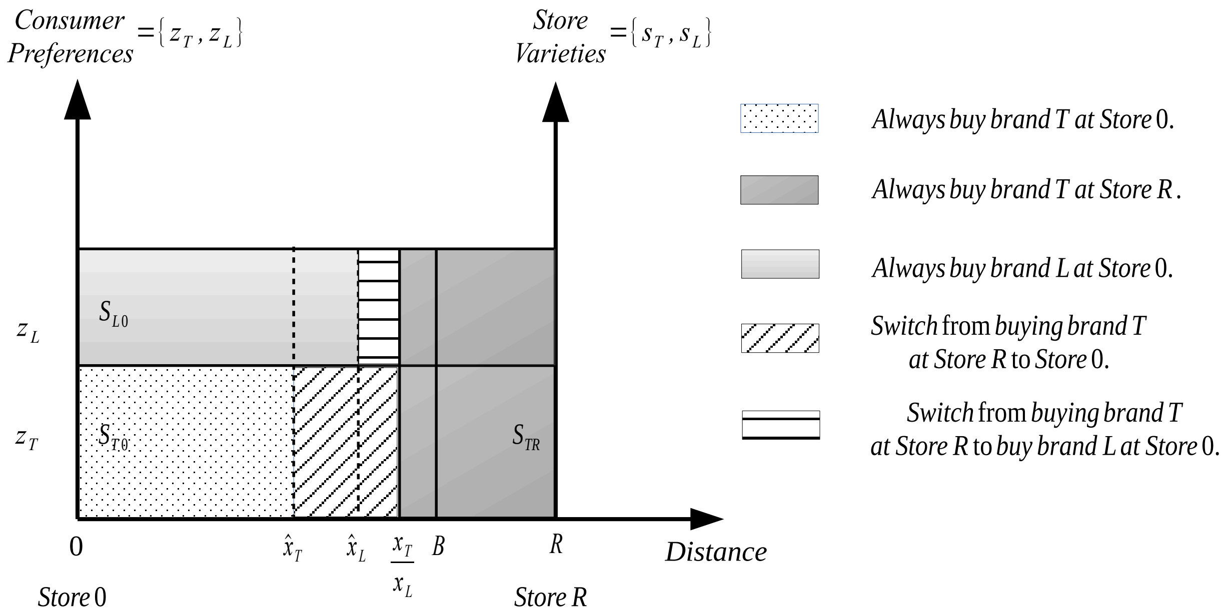

Figure 2: A Linear City with Varieties and Border

The indifferent "location" consumer x_T, who prefers the traded brand, has utility: $u-t\,|x_T-0|-p_{T0}=u-t\,|x_T-L|-p_{TR}-\beta$. This is the same problem we saw in Section 2, adding the $-\beta$ on the right side of the equality. Then, $x_T=\widehat{x}+\beta_T=\frac{p_{TR}-p_{T0}+tR}{2t}+\beta_T$. $\beta_T\in[0,\,(B-\widehat{x})]$ measure the number of consumers of product T that shit from store S_R to store S_0 due to the border cost β, and are represented by the 45-degree dashed lines in Figure 2. The indifferent "local" consumer x_L, the one that switches brands and stores, has utility: $u-t\,|x_L-0|-p_{L0}=u-t\,|x_L-R|-p_{TR}-\theta-\beta$, then $x_L=\widehat{x_L}+\beta_L=\frac{p_{TR}-p_{L0}+tR+\theta}{2t}+\beta_L$, where $\beta_L\in[0,\,(B-\widehat{x_L})]$ and $\beta_L\leq\beta_T$. β_L represents the number of L consumers who shift from buying brand T at store S_R to buying brand L at store S_0 due to the border, as represented by the horizontal lines in Figure 2. Additionally, $\widehat{x_L}$ represents the indifferent consumer between buying product L at store S_0 and product T at store S_R, as established previously, and is similar to $\widehat{x}$ for the traded product.

An additional restriction implies that, given a location, consumers should prefer to buy their preferred brand, ensuring each variety has a positive demand. That is, the consumer at 0 who prefers variety T should buy variety T, while the consumer at 0 who prefers variety L should buy variety L. The condition is $|p_{T0}-p_{L0}|<\theta$, that is, the price difference between brands at Store S_0 not being that large.[10].

[10]The consumer at 0 that prefer brand T has utility $u-t(0-0)-p_{T0}>u-t(0-0)-p_{L0}-\theta\implies(p_{T0}-p_{L0})<\theta$. On the other hand, a consumer at 0 that prefer brand L has utility $u-t(0-0)-$

Store S_0 sells the traded variety T and the local variety L, so its profits are $\pi_0 = p_{T0} \times (1-\lambda)x_T + p_{L0} \times \lambda x_L$ -2F $= p_{T0} \times (1-\lambda)(\frac{p_{TR}-p_{T0}+tR}{2t} + \beta_T) + p_{L0} \times \lambda(\frac{p_{TR}-p_{L0}+tR+\theta}{2t} + \beta_L) - 2F$. Maximizing in p_{T0} and p_{L0} we obtain $p_{T0} = \frac{p_{TR}+Rt+2t\beta_T}{2}$ and $p_{L0} = \frac{p_{TR}+tT+\theta+2t\beta_L}{2}$.

Store S_R sells only the traded variety T to both consumers, so its profits are $\pi_R = p_{TR} \times [(1-\lambda) \times (R - x_T) + \lambda \times (R - x_L)] - F$
$= p_{TR} \times \left[(1-\lambda)(R - (\frac{p_{TR}-p_{T0}+tR}{2t} + \beta_T)) + \lambda(R - (\frac{p_{TR}-p_{L0}+tR+\theta}{2t} + \beta_L))\right] - F$. Maximizing in p_{TR} we obtain $p_{TR} = \frac{(1-\lambda)p_{T0}+\lambda p_{L0}+Rt-\lambda\theta-2t[\beta_T+\lambda(\beta_L-\beta_T)]}{2}$.

Substituting reaction functions, we obtain:

$$p_{T0}^{BV} = tR - \frac{\lambda\theta}{6} + \frac{t\left[2\beta_T + \lambda\left(\beta_T - \beta_L\right)\right]}{3}, \tag{2}$$

$$p_{TR}^{BV} = tR - \frac{\lambda\theta}{3} - \frac{2t\left[\beta_T - \lambda\left(\beta_T - \beta_L\right)\right]}{3}. \tag{3}$$

For completion, $p_{L0}^{bv} = tR + \frac{(3-\lambda)\theta}{6} + \frac{t[(3-\lambda)\beta_L-(1-\lambda)\beta_T]}{3}$.[11] We now compute the size of the border by substituting prices into $x_T = \widehat{x} + \beta_T = \frac{p_{TR}-p_{T0}+tR}{2t} + \beta_T$ and $x_L = (R - \widehat{x}) + \frac{\theta}{2t} + \beta_L = \frac{p_{TR}-p_{L0}+tR+\theta}{2t} + \beta_L$. The equation for x_T has a solution $x_T = \frac{R}{2} - \frac{\lambda\theta}{12t} + \left(\frac{1}{3} + \frac{\lambda}{6}\right)\beta_T - \frac{\lambda}{6}\beta_L$, while the equation for x_L has a solution $x_L = \frac{R}{2} + \left(\frac{1}{2} - \frac{\lambda}{6}\right)\beta_L + \left(\frac{\lambda-1}{6}\right)\beta_T + \left(\frac{\theta}{4t} - \frac{\lambda\theta}{12t}\right)$. As $x_T \in \left(\frac{R}{2}, R\right)$ and $x_L \in \left(\frac{R}{2} + \frac{\theta}{2t}, R\right)$, we obtain $\beta_T \in \left(0, \frac{3R}{2}\right)$, and $\beta_L \in \left(0, \frac{3R}{2} - \frac{\theta}{2t}\right)$. Note that the border for the local product is always lower than that of the traded product. This is due to θ penalizing brand substitution and making the border less relevant for the local product.

The price difference is now:

$$(p_{T0}^{BV} - p_{TR}^{BV}) = \underbrace{\frac{(4-\lambda)}{3}t\beta_T}_{(I)} + \underbrace{\frac{\lambda\theta}{6}}_{(II)} + \underbrace{\frac{\lambda}{6}t\beta_L}_{(III)} \tag{4}$$

The next proposition shows the main result of the analysis.

Proposition 1. *The availability of local brands affects the estimation of the border effect.*

Proof. In Equation 4.

□

The term in (I) in Equation 4 is the adjusted border estimation. Previously, we showed in Lemma 2 that the border coefficient was $\frac{4}{3}t\beta$ higher than the term in (I). That is, the existence of local brands decreases the relevance of borders. As some consumers can switch to the local brand—within countries—less arbitrage is required between countries. It simply reflects the differences in preferences for traded goods between countries. In our model, λ is the share of consumers who prefer the local product over the traded product.

$p_{T0} - \theta < u - t(0-0) - p_{L0} \implies (p_{L0} - p_{T0}) < \theta$.

[11] Subtracting $p_{T0}^{BV} - p_{T0}^{BV} = \theta/2$ which meets the restriction $|p_{T0} - p_{T0}| < \theta$.

As some consumers in country 0 have access to the local brand, the impact of the border becomes less significant. If $\lambda = 0$, all consumers prefer the traded brand; then we are back to Lemma 2.

The term under (II) refers to the effect of substitutions for local brands, which decreases the market power of the traded brand in the local market. Due to the competition of local brands, the traded brand at location 0 needs to adjust its prices downwards. As shown in Borraz and Zipitría (2022), this effect is purely competitive and unrelated to the border. This is the effect of local competition that decreases market power at the country level: consumers of the traded product can switch to the local product. As λ—the share of consumers who prefer the local product—or θ—the cost of switching varieties—increases, the more significant the price difference of the traded product becomes.

The term under (III) reflects how easily consumers who prefer the local brand in country 0 can switch to the traded product *in country R*. Similar to term (I), it includes the transport cost t and the border costs β_L, which are the costs incurred by those who prefer local brands to switch to a different product or country. That is, the availability of a local brand on one side of the border affects the traded product on that side and on the *other side*. Consumers of the local brand have the same outside option as consumers of the traded brand: to cross the border and buy the traded product in another country (in our model, at L). This term is increasing in λ, t, and the border cost β_L.

Equation 4 decomposes the effect of border and local competition on price dispersion. The terms (I) and (III) are due to the border, but the term (II) is not. It could be debated whether the term in (III) is purely a border effect because the local brands' effect mediates, although it cannot be disentangled.

In Appendix A, we show that this result constitutes a Nash equilibrium. The results depend on several parameters; however, general conditions can be summarized in Table 1. Also, in the Appendix A, we show that the key for the equilibrium is that fixed costs F should be moderate, neither too high to Store S_0 dropping the local brand, nor too low to Store S_R also incorporating a local brand—i.e., all brands are traded—. The thresholds will be functions of the parameters of the model:

$$\underline{F}\left(R, t, \theta, \lambda, \beta, \beta_T, \beta_L\right) \leq F \leq \overline{F}\left(R, t, \theta, \lambda, \beta, \beta_T, \beta_L\right)$$

Nevertheless, the conditions for being a traded good are entirely based on F, not $\beta' s$. As we demonstrated in Section 2, traded products can still exist even when $\beta = \frac{3}{2}L$. Each country sells the traded brand to its local consumers in those cases. Border costs do not translate into local or traded products, but only into how many consumers buy from each store.

The following Section translates our theoretical model and the results under Equation 4 into a standard regression discontinuity design. We discuss the exogeneity assumption of the border and how it relates in empirical papers to the evidence of local brands.

Parameter	**Desired Condition**	**Rationale**
Market size (R)	Large	Scales quadratic and linear profit components; increases the value of variety and segmentation.
Local preference share (λ)	Balanced (e.g., $0.4 < \lambda < 0.6$)	Ensures balanced market access and profit gains from both consumer segments.
Transport cost (t)	Moderate	Very low t weakens market power; very high t limits access. Optimal differentiation occurs at moderate t.
Brand switching cost (θ)	Moderate	Supports product segmentation without excessive exclusion, avoiding profit erosion from high switching costs.
Benchmark border cost (β)	Moderate to high	Increases disutility in the benchmark; lowers benchmark profits, enhancing relative gain from a richer model.
Traded product border cost (β_T)	Moderate to high (within $\left(0, \frac{3R}{2}\right)$)	Boosts differentiation and interaction effects in the richer model.
Local product border cost (β_L)	Close to β_T (within $\left(0, \frac{3R}{2} - \frac{\theta}{2t}\right)$)	Enhances brand loyalty and strengthens pricing power.

Table 1: General parameter conditions for a Nash equilibrium.

4 Econometric Analysis

Most of the papers' empirical analyses applied non-parametric sharp regression discontinuity designs to estimate the effect of borders on traded products. At a distance small enough on both sides of the border, everything should be equal except the price of the traded goods. Then, the border causes prices to diverge, and the effect can be measured. A fundamental requirement of RDD is that all variables, other than the treatment, exhibit continuity at the cutoff. However, the data in all the papers reviewed in section 1 showed that this is not the case: most products are local, that is, are on one side or the other, but not on both. Assume the impact of the border on prices is measured according to the following equation:

$$p_i^h = \alpha_i + \beta B + \theta D^h + \psi D^h B + \gamma^h X^h + \varepsilon_i^h, \quad (5)$$

Where i is for good, h is the unit of analysis (store as in Gopinath, Gourinchas, Hsieh, and Li (2011), or household as in Beck, Kotz, and Zabelina (2020)) B is a dummy variable that takes the value one if the store or household is not in the reference country, say in Canada vs. being in the US, D^h is the distance of the unit of analysis—store, household—to the border, and X^h are observable characteristics of the unit of analysis—income, age, size of the store, etc.—. Note the similarity between the paper's empirical equation and our theoretical model and how the model can adopt an empirical estimation strategy. The parameter of interest is β, and the identifying assumption is that the error term ε_i^h is uncorrelated with the border dummy variable B, i.e., $E[\varepsilon_i^h|B] = 0$.

In Section 3, we showed that the prices of traded products are affected by local products. We incorporate local products into Equation 5 by adding two terms: X_{ij}^{h-} for product i at unit h at the left of the border $h-$, and X_{ij}^{h} for product i at unit h at the right of the border h, the convention for the reference country, and where j refers to local products in the same product category of product i. As shown in Section 3, the existence of local products is independent of border costs and is determined by fixed costs. As a result, they are uncorrelated with the border B.

Noting that X_{ij}^{h-} and X_{ij}^{h} are each one only on one side of the border, they can be written as $X_{ij}^{h-}(1-B)$ and $X_{ij}^{h}B$, we add both terms to Equation 5 and rewrite it as:

$$p_i^h = \alpha_i + \left[\beta + \left(\delta_1 X_{ij}^h - \delta_2 X_{ij}^{h-}\right)\right] B + \delta_2 X_{ij}^{h-} \theta D^h + \psi D^h B + \gamma^h X^h + u_i^h, \tag{6}$$

Two conclusions can be reached from Equations 5 and 6. First, ε_i^h in Equation 5 is correlated with B due to the omitted variable bias. Formally, $\varepsilon_i^h = \left(\delta_1 X_{ij}^h - \delta_2 X_{ij}^{h-}\right) B + u_i^h$, which is correlated with the border. Secondly, we can rearrange Equation 6 to isolate the effect of local varieties from the estimation of the border, as with the interaction of the distance parameter:

$$p_i^h = \alpha_i + \beta \underbrace{B}_{(I)} + \delta_2 \underbrace{X_{ij}^{h-}}_{(II)} + (\delta_1 - \delta_2) \underbrace{\left(X_{ij}^h - X_{ij}^{h-}\right) B}_{(III)} + \theta D^h + \psi D^h B + \gamma^h X^h + u_i^h, \tag{7}$$

The underset numbers mirror those in Equation 4. Although Equation 4 is in differences and Equation 7 is in levels, as the terms (I) to (III) are turned on and off depending on whether we are on the right or left of the border, it mimics an equation in differences. The omission to control for local brand bias can lead to a biased estimation of the border effect through terms (II) and (III). Both are required to estimate the impact of the border on price dispersion.

No paper has previously addressed the issue arising due to local products in price dispersion across countries. Within countries, without taking into account the border effect, we have estimated the effect of having one product difference between two stores to account for up to 0.8% of the price differences. The counting of products—or UPC codes—between locations has also been used by (Cavallo, Feenstra, and Inklaar, 2023) for showing that differences in varieties with the US are the primary source of explanation for the increased cost of living. While this is a first approximation, the measure will assign the same value to very different competitive settings: a product that is hardly sold will be valued the same as a highly selling brand and, thus, will have the same effect on price dispersion.

Alternatively, in Borraz and Zipitría (2024), we analyze both short- and long-run price dispersion and use an entropy index to measure product differences within a given product category across markets. In this setting, an entropy index measures the sparsity of products between markets and increases as the number of countries offering different varieties increases, mainly when local brands are prevalent. Additionally, since the border variable is a dummy, there will be no collinearity problems between the two variables.

Lastly, Auer, Burstein, and Lein (2021) proposed a method to correct the impact of traded products on the exchange rate pass-through in Switzerland. They provide shares of the proportion of traded products to traded and local products by categories to control for the impact of the appreciation of the Swiss Franc in 2015. This approach could also be used to correct the presence of local products when estimating the effect of borders on prices and to isolate their impact.

The corrections will depend on the information available. For example, Auer, Burstein, and Lein (2021) has information on purchases but not on other products not consumed at the store. This limits the possibility of controlling for actual restrictions on consumer purchase options and store pricing decisions. On the other hand, Borraz and Zipitría (2022) has information on prices for products sold at the same store but not actual purchases. While this information is suitable for better controlling local brands, when complemented with purchase data, it will be challenging to weigh the relevance of each product (θ).

5 Conclusions

The paper aims to clarify the role of borders, their impact on prices, and product entry decisions. We define two products: traded ones are sold in more than one country, and local ones are sold only in one country. Within-country fixed costs explain whether products are traded or sold in two countries. After showing the isolated effect of borders on price dispersion, we show how local products affect the estimation of the border effect. We decompose the dispersion of prices into three components: one is the adjusted border estimation of the traded product, a second effect reflects the impact of local products on the price of traded products, and the third one is a border cost related to local products. This result differs from the benchmark border estimation.

Then, we show how RDD are affected by local products, which are not continue at the cutoff. This biases the estimation of the border effect. Lastly, we propose measures from other papers that can be adapted to provide an unbiased estimation of the border effect. In all, we conclude that the impact of borders on price dispersion is overestimated. Borders seem to have a much smaller role, due to the effect of differences in competition between countries explaining a non-negligible share of the border effect.

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A Appendix: Nash Equilibrium in Section 3

In Section 3, we found that the store at 0 sells two products, one local (B) and one traded (B). In contrast, the Store S_L sells only the traded product A, and vice versa. There are several alternative scenarios, of which we will consider just two. First, the store S_0 could withdraw the local product, and only the traded brand A is sold in both stores. This alternative scenario is interesting because it allows us to demonstrate that firms benefit from selling local brands. Second, Store S_L also introduces a local brand, creating a symmetrical scenario between the two stores. A less interesting scenario is that both stores sold only local products, they specialize, and the model has no traded brands.

The key to showing the results is the fixed costs F that stores must pay to sell the products. If F is too high, selling two products will be unprofitable; if F is too low, stores will add infinite products. As a result, F will be bound below and above.

A.1 Store 0 Withdraw the Local Product

In Section 3 we calculate the prices of the products: $p_{T0}^{BV} = tR - \frac{\lambda\theta}{6} + \frac{t[2\beta_T + \lambda(\beta_T - \beta_L)]}{3}$, $p_{TR}^{BV} = tR - \frac{\lambda\theta}{3} - \frac{2t[\beta_T - \lambda(\beta_T - \beta_L)]}{3}$, and $p_{L0}^{bv} = tR + \frac{(3-\lambda)\theta}{6} + \frac{t[(3-\lambda)\beta_L - (1-\lambda)\beta_T]}{3}$. Benefits are $\pi_0 = p_{T0} \times (1-\lambda)(\frac{p_{TR} - p_{T0} + tR}{2t} + \beta_T) + p_{L0} \times \lambda(\frac{p_{TR} - p_{L0} + tR + \theta}{2t} + \beta_L) - 2F$ and $\pi_R = p_{TR} \times \Big[(1-\lambda)(R - (\frac{p_{TR} - p_{T0} + tR}{2t} + \beta_T)) + \lambda(R - (\frac{p_{TR} - p_{L0} + tR + \theta}{2t} + \beta_L))\Big] - F$.

Solving the system,[12] we have:

$$\begin{aligned}
\pi_0 = \frac{1}{72t}\Big[&- 5\lambda^2\theta^2 + 9\lambda\theta^2 \\
&+ 4t\Big(9R^2t + 12Rt\beta_L\lambda - 12Rt\beta_T\lambda + 12Rt\beta_T + 6R\lambda\theta \\
&\quad - 5\lambda^2 t\beta_L^2 + 9\lambda t\beta_L^2 + 10\lambda^2 t\beta_L\beta_T - 10\lambda t\beta_L\beta_T \\
&\quad - 5\lambda^2\theta\beta_L + 9\lambda\theta\beta_L - 5\lambda^2 t\beta_T^2 + \lambda t\beta_T^2 + 4t\beta_T^2 \\
&\quad + 5\lambda^2\theta\beta_T - 5\lambda\theta\beta_T\Big)\Big] - 2F
\end{aligned} \tag{8}$$

$$\begin{aligned}
\pi_R = \frac{1}{18t}\Big[&\lambda^2\theta^2 \\
&+ t\Big(9R^2t - 12Rt\beta_L\lambda + 12Rt\beta_T\lambda - 12Rt\beta_T - 6R\lambda\theta \\
&\quad + 4\lambda^2 t\beta_L^2 - 8\lambda^2 t\beta_L\beta_T + 8\lambda t\beta_L\beta_T + 4\lambda^2\theta\beta_L \\
&\quad + 4\lambda^2 t\beta_T^2 - 8\lambda t\beta_T^2 + 4t\beta_T^2 - 4\lambda^2\theta\beta_T + 4\lambda\theta\beta_T\Big)\Big] - F
\end{aligned} \tag{9}$$

[12] All calculations and tables were made using OpenAI (2023), using SymPy.

In Section 2, we calculate the equilibrium prices when stores sell the international brand and there are borders $p_0^b = tL + \frac{2t\beta}{3}$ and $p_L^b = tL - \frac{2t\beta}{3}$. Substituting into profits $\pi_0^b = \left(\frac{p_L - p_0 + tL + 2t\beta}{2t}\right) \times p_0 - F$ and $\pi_L^b = \left(\frac{p_0 - p_L + tL - 2t\beta}{2t}\right) \times p_L - F$, we obtain the profits when there is only one traded brand:

$$\pi_0^B = \frac{tR^2}{2} - \frac{2t\beta^2}{9} - F$$
$$\pi_R^B = \frac{tR^2}{2} - \frac{2tR\beta}{3} + \frac{2t\beta^2}{9} - F$$

Given that all parameters are positive, $\lambda \in (0, 1)$, and $\beta_T > \beta_L$, we need to prove the parametric conditions under which $\pi_0 > \pi_0^B$ and $\pi_R > \pi_R^B$. The exact algebraic results are:

$$\begin{aligned}
\pi_0 - \pi_0^B = & -F + \frac{2}{3}Rt\left(\beta_L\lambda - \beta_T\lambda + \beta_T\right) + \frac{1}{3}R\lambda\theta + \frac{2t\beta^2}{9} \\
& + \frac{t}{18}\left(-5\lambda^2\beta_L^2 + 9\lambda\beta_L^2 + 10\lambda^2\beta_L\beta_T - 10\lambda\beta_L\beta_T - 5\lambda^2\beta_T^2 + \lambda\beta_T^2 + 4\beta_T^2\right) \\
& + \frac{1}{18}\left(-5\lambda^2\beta_L\theta + 9\lambda\beta_L\theta + 5\lambda^2\beta_T\theta - 5\lambda\beta_T\theta\right) + \frac{1}{72t}\left(-5\lambda^2\theta^2 + 9\lambda\theta^2\right)
\end{aligned} \tag{10}$$

$$\begin{aligned}
\pi_R - \pi_R^B = \frac{1}{18t}\Big[& \lambda^2\theta^2 + t\Big(18F - 9R^2t + 12Rt\beta - 4t\beta^2 \\
& - 18F + 9R^2t - 12Rt\beta_L\lambda + 12Rt\beta_T\lambda - 12Rt\beta_T - 6R\lambda\theta \\
& + 4\lambda^2t\beta_L^2 - 8\lambda^2t\beta_L\beta_T + 8\lambda t\beta_L\beta_T + 4\lambda^2\theta\beta_L \\
& + 4\lambda^2t\beta_T^2 - 8\lambda t\beta_T^2 + 4t\beta_T^2 - 4\lambda^2\theta\beta_T + 4\lambda\theta\beta_T\Big)\Big]
\end{aligned} \tag{11}$$

From equation $\pi_0 - \pi_0^b$ we obtain the condition on F for that expression to be positive:

$$\begin{aligned}
F < \frac{1}{72t}\Big[& 48R\beta_L\lambda t^2 - 48R\beta_T\lambda t^2 + 48R\beta_T t^2 + 24R\lambda t\theta + 16t^2\beta^2 \\
& - 20t^2\lambda^2\beta_L^2 + 36t^2\lambda\beta_L^2 + 40t^2\lambda^2\beta_L\beta_T - 40t^2\lambda\beta_L\beta_T \\
& - 20t\lambda^2\beta_L\theta + 36t\lambda\beta_L\theta - 20t^2\lambda^2\beta_T^2 + 4t^2\lambda\beta_T^2 + 16t^2\beta_T^2 \\
& + 20t\lambda^2\beta_T\theta - 20t\lambda\beta_T\theta - 5\lambda^2\theta^2 + 9\lambda\theta^2\Big]
\end{aligned} \tag{12}$$

The general parameter conditions for both expressions to be positive were shown in Table 1 in Section 3.

A.2 Store L incorporates the Local Product (both products are traded)

The alternative to Store 0, with withdrawing the local brand, is to Store L to incorporate one. Now both stores sell brands T and L, although we maintain their names. This is the same analysis of Section 2, but with two brands. While now there are four demands and prices, the general result does not change, as both problems are symmetrical. Remember that the condition $|p_T - p_L| < \theta$ guaranties that each product will have (all) demand; i.e., consumers buy their preferred brand in equilibrium or one product is out of the market. The only difference is that both benefits have two fixed costs.

Solving the model with the data in Section 2, we obtain $D_0 = \frac{R}{2} + \frac{\beta}{3}$ and $D_R = \frac{R}{2} - \frac{\beta}{3}$. Then, profits are (subtracting F for both products): $\pi_0^{2B} = \left(\frac{R}{2} + \frac{\beta}{3}\right) \times \left(tR + \frac{2t\beta}{3}\right) - 2F$ and $\pi_R^{2b} = \left(\frac{R}{2} - \frac{\beta}{3}\right) \times \left(tR - \frac{2t\beta}{3}\right) - 2F$.

For store S_0, fixed costs are always duplicated, but for store S_R, not. So we have one cutoff for $\pi_R - \pi_R^{2B} > 0$ that is that F should be high enough:

$$\begin{aligned} F > \frac{1}{18t}\Big[&12R\beta t^2 - 12R\beta_L\lambda t^2 + 12R\beta_T\lambda t^2 - 12R\beta_T t^2 - 6R\lambda t\theta - 4\beta^2 t^2 \\ &+ 4\beta_L^2\lambda^2 t^2 - 8\beta_L\beta_T\lambda^2 t^2 + 8\beta_L\beta_T\lambda t^2 + 4\beta_L\lambda^2 t\theta \\ &+ 4\beta_T^2\lambda^2 t^2 - 8\beta_T^2\lambda t^2 + 4\beta_T^2 t^2 - 4\beta_T\lambda^2 t\theta + 4\beta_T\lambda t\theta + \lambda^2\theta^2\Big] \end{aligned} \tag{13}$$

The inequality $\pi_0 - \pi_o^{2b} > 0$ is fulfilled for the parameter conditions, as the next Theorem shows.

Theorem. Let $t > 0$, $R > 0$, and $\theta \geq 0$. Assume the parameters satisfy:

$$\lambda \in (0,1), \quad \beta \in \left[0, \frac{3R}{2}\right], \quad \beta_T \in \left(0, \frac{3R}{2}\right), \quad \beta_L \in \left(0, \frac{3R}{2} - \frac{\theta}{2t}\right), \quad \beta_L < \beta_T$$

Then the profit difference between the richer model and the two-border benchmark at location 0 satisfies:

$$\pi_0 - \pi_0^{2B} > 0$$

Proof. We analyze the expression:

$$\begin{aligned} \pi_0 - \pi_0^{2B} = \frac{1}{72t}\Big[&-5\lambda^2\theta^2 + 9\lambda\theta^2 + (\text{linear terms in } \theta) \\ &+ (\text{quadratic and interaction terms in } \beta_T, \beta_L) \\ &+ (\text{market size terms in } R) - 4t^2(3R + 2\beta)^2\Big] \end{aligned}$$

The leading term in θ^2 simplifies to $\lambda(9-5\lambda)\theta^2$, which is strictly positive for all $\lambda \in (0,1)$. The linear terms in θ are positive due to $\beta_T > \beta_L$, and the interaction terms involving β_T^2, β_L^2, and $R\beta$ are also positive under the stated bounds.

Although the benchmark includes a large negative quadratic term in $(3R+2\beta)^2$, this is dominated by the quadratic gain in R^2 from the full model and the large number of additive positive border-cost and interaction effects.

Hence, the overall expression is strictly positive under the assumed parameter bounds.

□

A.3 Summarizing

In the previous two sections, we found general conditions for having a local brand to be a Nash equilibrium. The more demanding conditions are those in Section A.1, which have general parameter conditions summarized in Table 1. There are also conditions on the fixed cost F. If two low, Equation 13 says that Store S_L will also incorporate the local brand. If two high, Equation 12 says that Store S_0 will prefer not to sell the local brand. As a result, fixed costs need to be intermediate.

One last issue is if both stores specialize in one local product. While this is a plausible scenario, it is less interesting for the paper. At the same time, it is an issue in itself, as there are different possibilities. First, stores can specialize in one product, say Store S_0 in product A and Store S_L in product B, and set the price to sell to *all* consumers in the product variety, regardless of the country. Second, it could be that the stores sell to *all* consumers in the country, regardless of variety. Third, stores could maximize profits by selling to some consumers, leaving those further apart not consuming.

We skip this lengthy demonstration, as our two previous settings explain that, for some parameter combination, a Nash equilibrium supports local and traded goods, consistent with the empirical evidence. On the contrary, this also shows that if those parameters do not hold, then other results are possible.